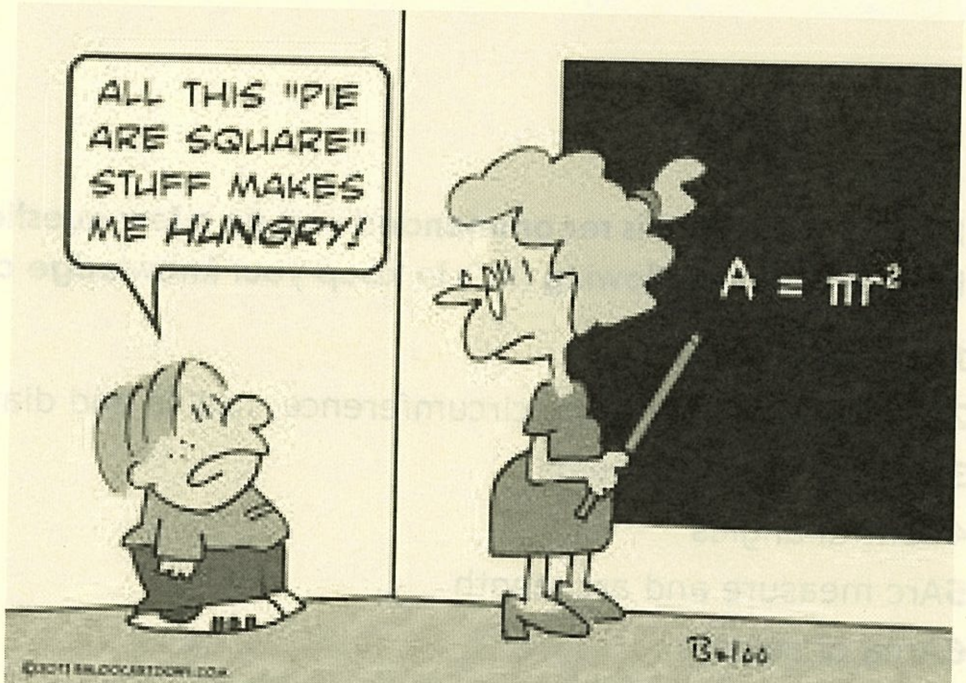
Math 9

Unit 8: Circle Geometry



Name KEY

Class________

Outcomes:

1. Solve problems and justify the solution, using the following circle properties:
- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency.

IXL's for this unit- It is recommended you do a few questions a night from one of the following IXL's to keep your knowledge current!

F.1 Parts of a circle

F.2 Circles: calculate area, circumference, radius and diameter

F.3 Circles: word problems

F.4 Central angles

F.5 Arc measure and arc length

F.6 Area of sectors

F.7 Circle measurements: mixed review

F.8 Arcs and chords

F.9 Tangent lines

F.10 Perimeter of polygons with an inscribed circle

F.11 Inscribed angles

F.12 Angles in inscribed right triangles

F.13 Angles in inscribed quadrilaterals

9.0- Into: review and terms

Types of angles

1. **Acute angles**... less than 90°

2. **Right angles**... Exactly 90°

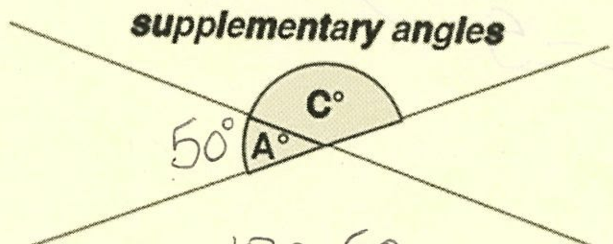
3. **Obtuse angles**... over 90°. Less than 180°

4. **Reflex angles**... over 180°. Less than 360°

Properties of Angles

1. **Straight Angle** – Exactly 180°

2. Any angles (two or more) that add up to 180 degrees are also called



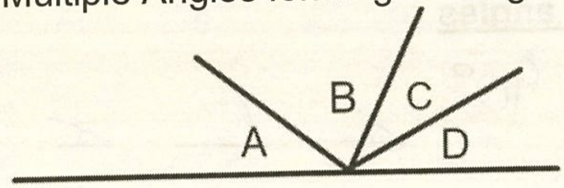
180-50

If angle A is 50 degrees, angle C must be 130° degrees.

If angle A were 35 degrees, angle C must be 145° degrees.

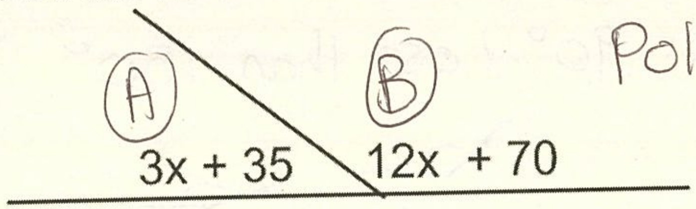
180-35

Multiple Angles forming a straight angle



Angles A + B + C + D = 180° degrees.

How can we solve for unknown angles with variables?



Polynomial rules.
combine like terms

$3x+35+12x+70=180$

$15x+105=180$
$-105 \quad -105$

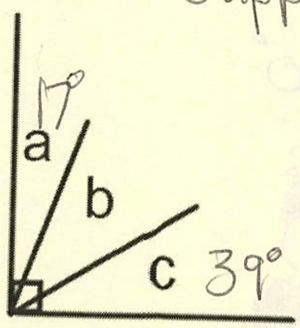
$\frac{15x}{15}=\frac{75}{15}$

$x=5$

$\angle A$
$3(5)+35$
$15+35$
$\boxed{50^\circ}$

$\angle B$
$180-50$
$\boxed{130^\circ}$

3. Any angles (two or more) that add up to 90 degrees are also called Supplementary angles

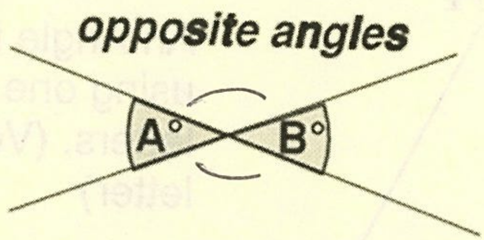


$a + b + c = 90^\circ$

If $a = 17^\circ$ and $c = 39^\circ$, then $b =$ <u>34°</u>

90-17-39=

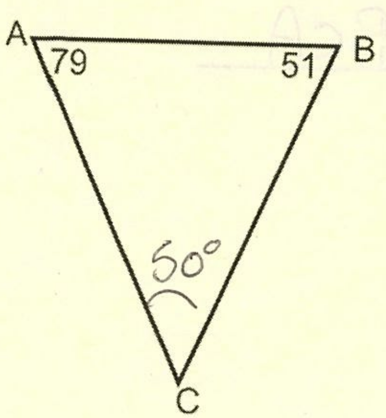
4. **<u>Opposite angles</u>**... are two angles formed by intersecting lines. They share a vertex. Opposite angles are <u>congruent</u>→ Means the Same



If angle A is 50 degrees, angle b must be <u>50°</u> degrees.
The other 2 angles must measure <u>130°</u> degrees.
180-50=130°

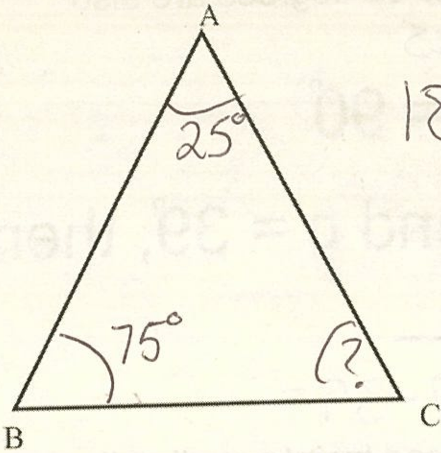
If angle B is 35 degrees, angle A must be <u>35°</u> degrees.
The angle that forms a linear pair with angle A must be <u>145°</u> degrees.

5. The interior angles of a triangle must add up to 180 degrees



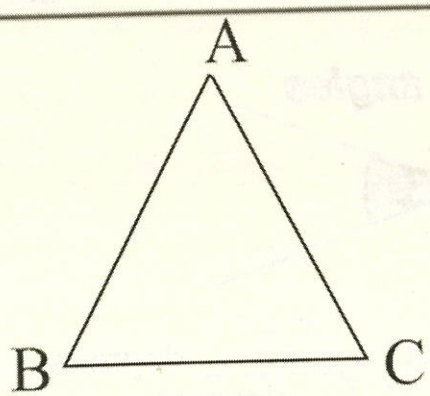
180-79-51=50°

Find the measure of angle C, if angle A is 25, and angle B is 75 degrees



$180 - 25 - 75 = 80°$

How to name an angle



An Angle in a triangle can be named using one letter (vertex) or by three letters. (Vertex must be the middle letter)

For example, < A = < BAC = < CAB

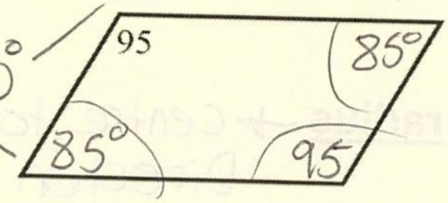
Likewise, < B = ∠ABC = ∠CBA

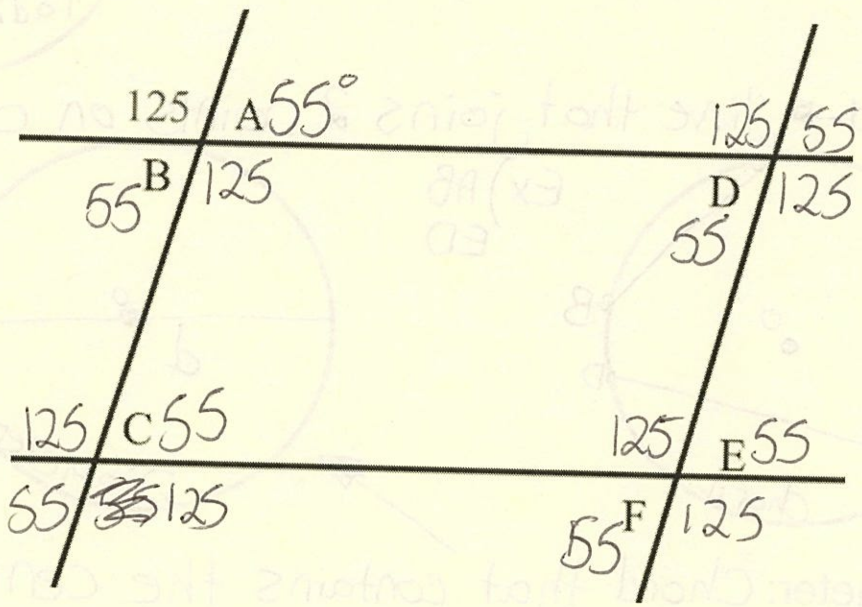
And < C = ∠ACB = ∠BCA

Parallelograms:
Opposite angles will be Congruent → Same

= 180°

∠ equals 360°

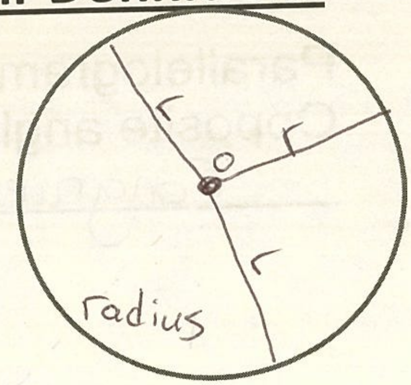




Worksheet: Circle Geometry Intro: Working with angles

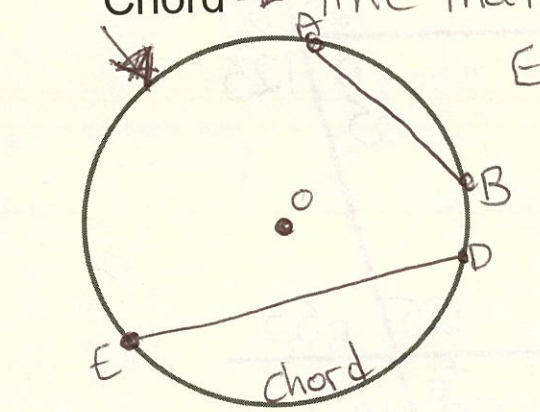
9.0b Properties of Circles & Their Definitions

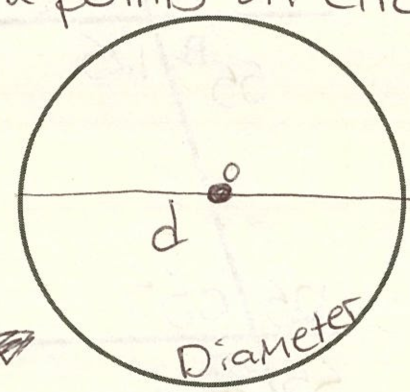
A **radius** → Centre to rim. Any Direction



Chord → line that joins 2 points on circle

Ex) AB
ED

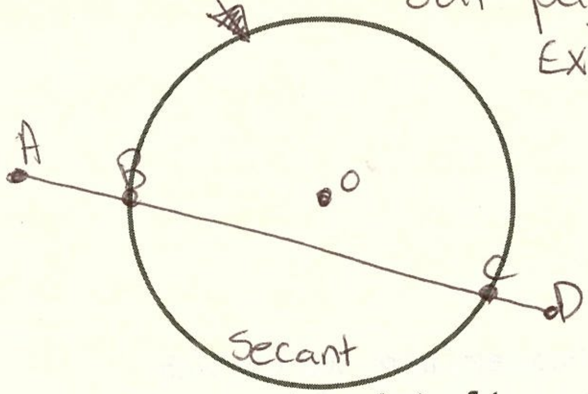


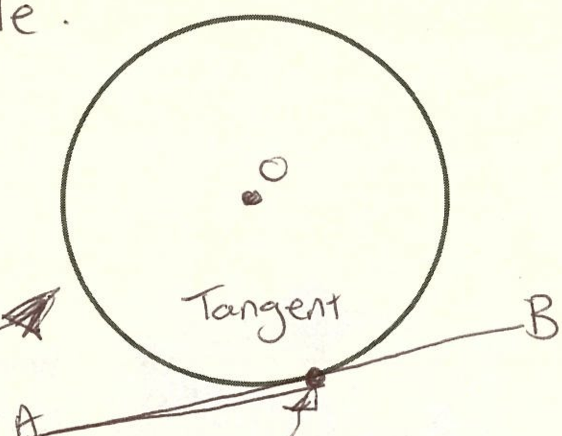


Diameter: Chord that contains the centre.

Secant: Intersects in exactly 2 points. Sticks out past circle.

Ex) ~~AB~~
AD





Tangent (point of tangency):

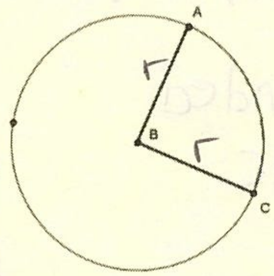
Touches circle at only one point

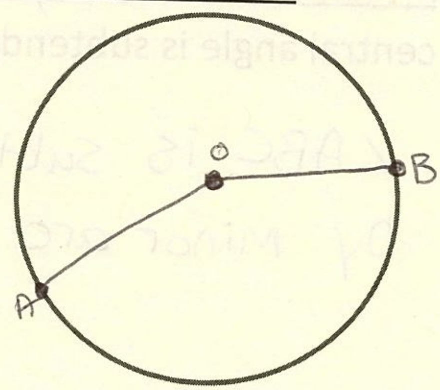
Central Angle

pointy part

An angle whose Vertex is at the Center of a circle.

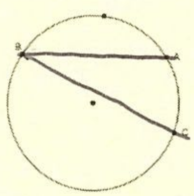
* notice the arms of a central angle are radii

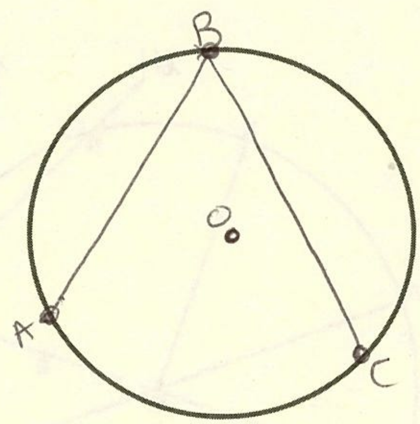




Inscribed Angle

An angle whose Vertex is on the Circle, and the two sides make a Chord.



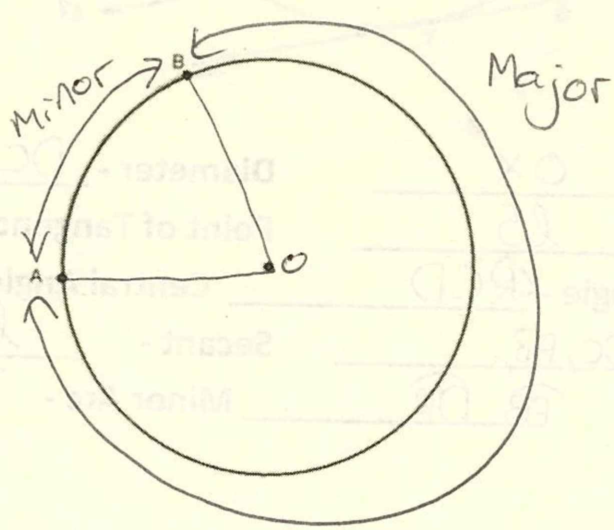


Arc

Circumference of a circle that connects two points. Major arcs and minor arcs.

Minor
<180° of circle

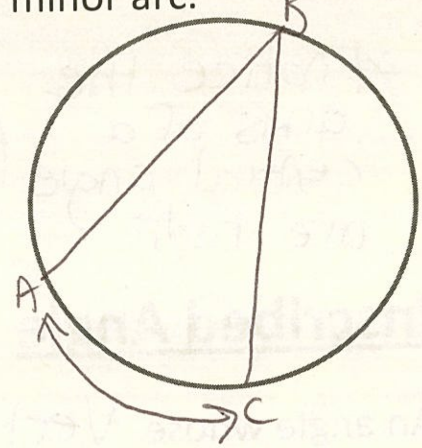
Major
$\geq$ 180° of circle



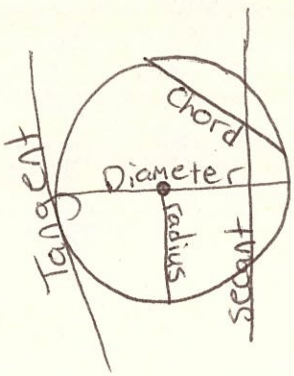
Subtended

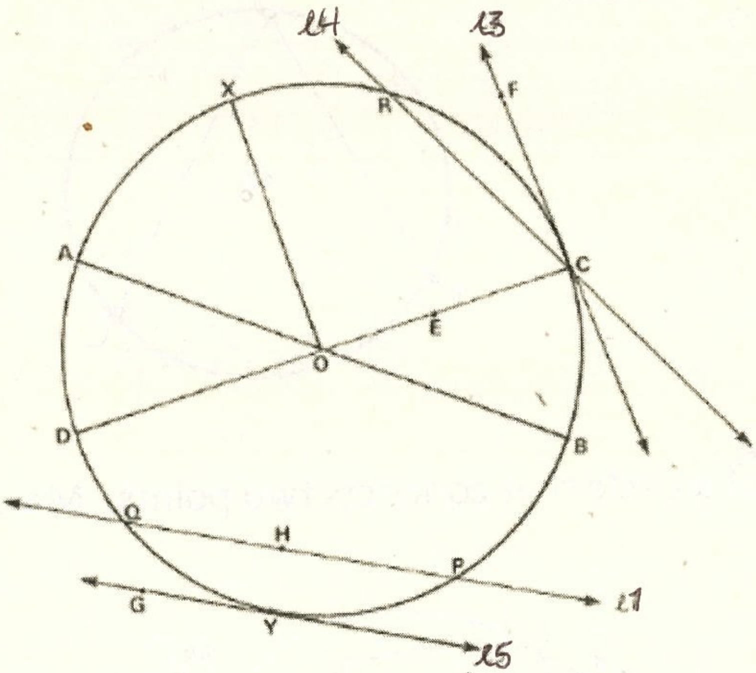
Means Combined, or found on. Ex) an inscribed or central angle is subtended by a major or minor arc.

∠ABC is subtended by minor arc AC



Label each Circle Property





Radius - OX

Diameter - DC, AB

Tangent - ℓ5

Point of Tangency - Y, C

Inscribed Angle - ∠RCD

Central Angle - ∠COB, ∠DOX

Chord - DC, AB,

Secant - ℓ1

Major Arc - $\overset{\frown}{AB}$ $\overset{\frown}{DR}$

Minor Arc - $\overset{\frown}{BC}$, $\overset{\frown}{AD}$

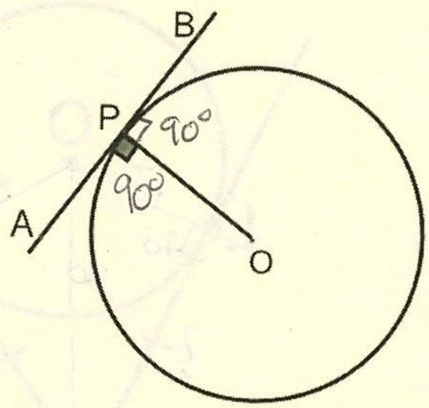
9.1 Tangent Properties

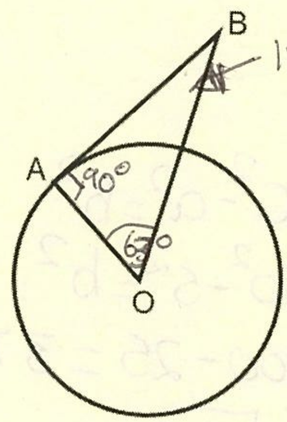
Tangent Property #1:

$\perp$ 90°

Tangent to a circle is perpendicular to the radius at the point of tangency.

→ **RULE:** A tangent to a circle creates a 90° angle with the radius at the point of tangency



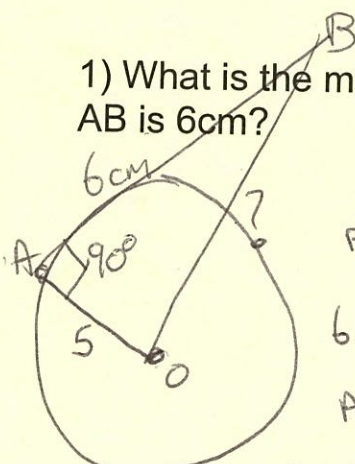


$180-90-63$
$=27^\circ$

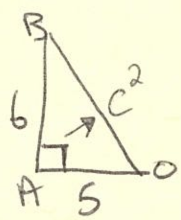
** We use this property to create right triangles. Then we can use Pythagorean theorum to solve for side lengths.

Formula: $a^2+b^2=c^2$ $\qquad c^2-a^2=b^2$

1) What is the measure of BO if the diameter of the circle is 10cm and AB is 6cm?



radius = 5cm



$a^2+b^2=c^2$

$6^2+5^2=c^2$

$36+25=61$

$\sqrt{61}=$

$\boxed{7.81\text{cm}}$

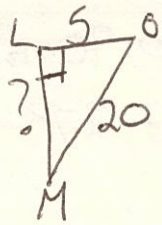
*Side across from right angle must be 90°

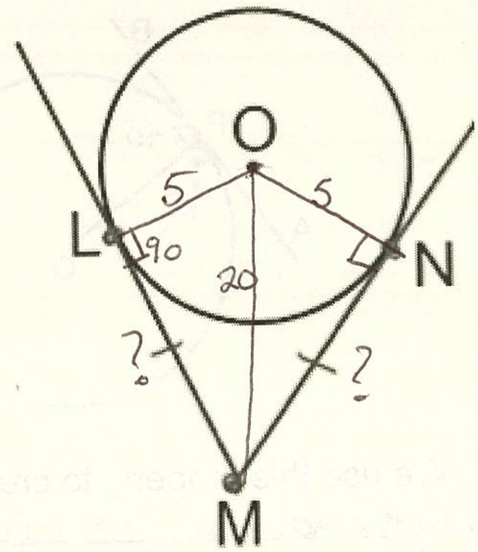
Tangent Property #2:

Two tangent lines form a common point outside a circle

Two tangent lines can be drawn from a common point outside of the circle. They will be equal distance from the points of tangency.

1) If the radius is 5cm and the distance from the origin (centre) to point M is 20cm, can we determine the length of each tangent line to point M?





$c^2 - a^2 = b^2$

$20^2 - 5^2 = b^2$

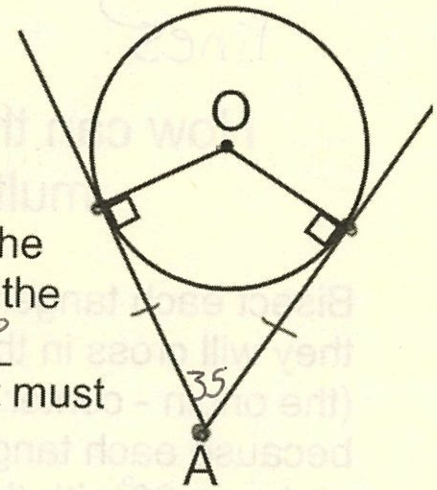
$400 - 25 = 375$

$\sqrt{375}$

$\boxed{19.36\text{cm}}$

Combining Tangent Properties 1 & 2:

When combining tangent properties 1 & 2,we can conclude then, that the two tangent lines meet two radii there is now a 4-sided object formed with two 90° angles at the points of tangency. Because of this, the other two angles must add to 180° since the degrees in a 4-sided object must add up to a total of 360°.



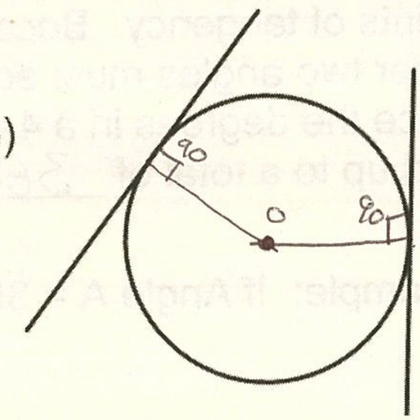
Example: If Angle A = 35°, then Angle O = 145°

180-35=

Tangent Property #3: Centre Locating the ~~origin~~ using tangent lines.

How can the origin be located using multiple tangent lines?

Bisect each tangent line - they will cross in the middle (the origin - center of the circle) because each tangent line makes a 90° with the radius (property #1).

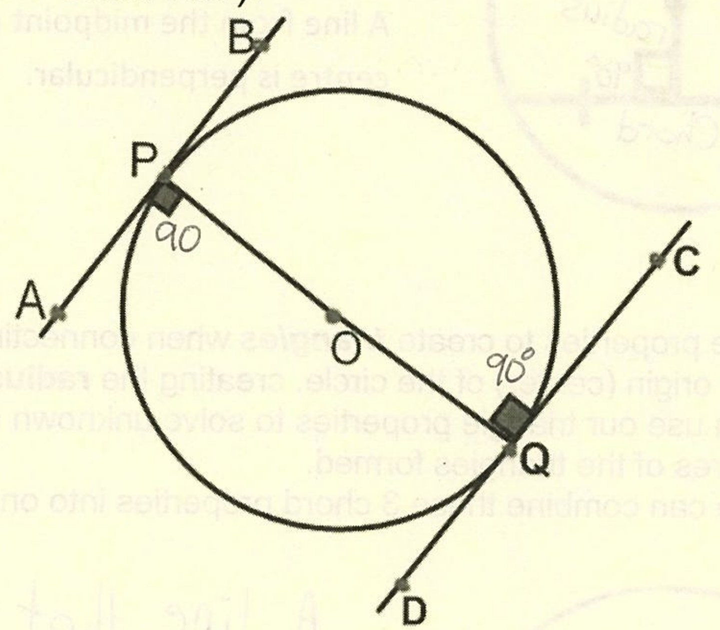


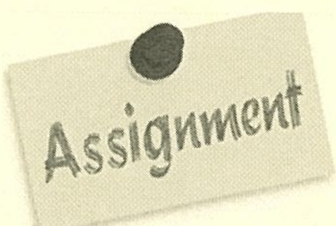
Tangent Property #4:

Parallel tangent lines

When are two tangent lines parallel?

Tangent lines are parallel when they use opposite ends of the diameter as points of tangency (so… when they intersect a circle at opposite endpoints of a diameter).



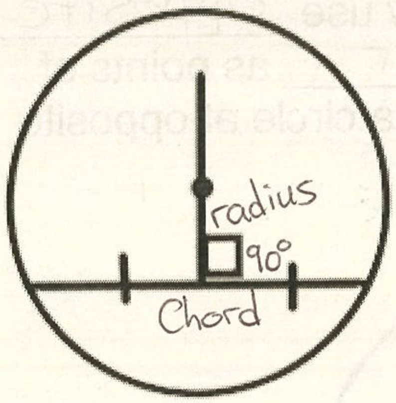


Pythagorean Theorem worksheet

IXL F.9 Tangent Lines

9.2 Chord Properties

Chord properties:



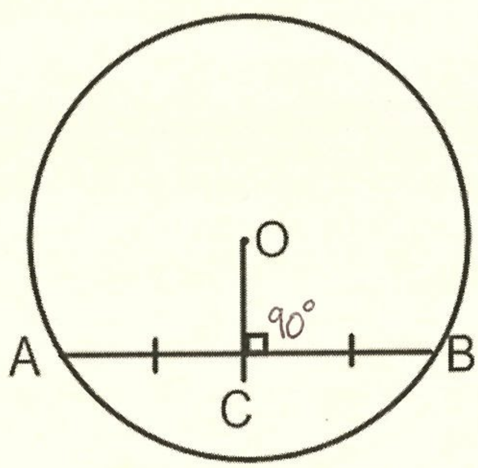
A line from the center will bisect a chord if it is perpendicular. (90°)

And vice versa

A line from the midpoint of a chord to the centre is perpendicular.

We use these properties to create ***triangles*** when connecting end points of chords to the origin (center) of the circle, creating the **radius** of the circle. Then, we can use our triangle properties to solve unknown side lengths & angle measures of the triangles formed.

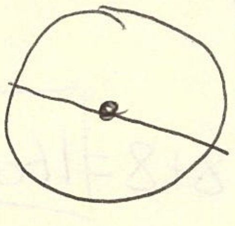
We can combine these 3 chord properties into one rule:



A line that perpendiculary 90°
(cut in 2)
bisects a chord, passes through the origin.

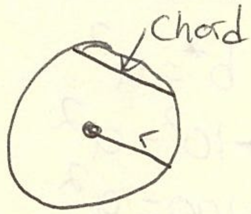
Something to Think About...
(Apply Chord Properties)

1. Can a chord be longer than a diameter? Yes or No
   Why or Why not?



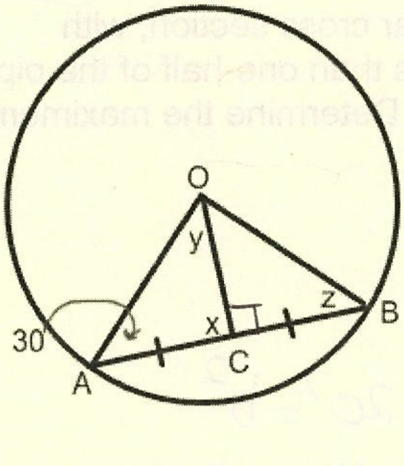
No. Diameter is the longest chord

2. Can a chord be shorter than a radius? Yes or No
   Why or why not?



Yes.

Example #1: Determine the measures of Angles *x*, *y*, and *z*.

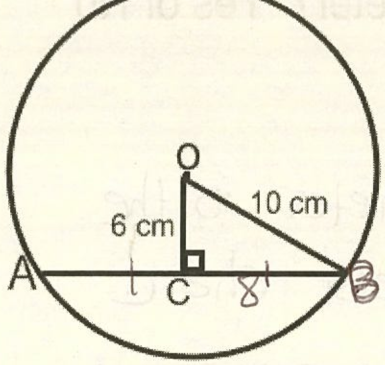


$x = 90°$

$y = 60 \rightarrow 180 - 90 - 30 = 60°$

$z = 30°$

Example #2: O is the center of the circle. Find the length of chord AB.



$c^2 - b^2 = a^2$

$10^2 - 6^2 = a^2$

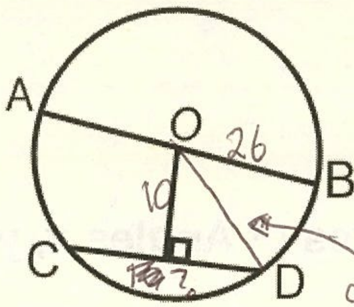
$100 - 36 = 64$

$\sqrt{64}$

$\boxed{8}$

$8 + 8 = \boxed{16cm}$

Example #3: AB is a diameter with length 26 cm. CD is a chord that is 10 cm from the center O. What is the length of the chord?



draw another radii

Diameter=26
radius=13

$c^2 - b^2 = a^2$

$13^2 - 10^2 = a^2$

$169 - 100 = a^2$

$a^2 = 69$

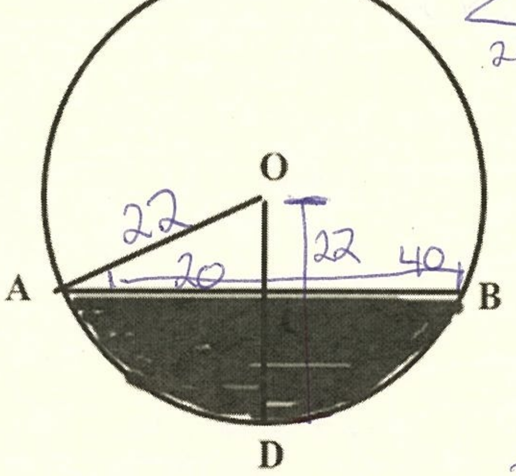
$\sqrt{69}$

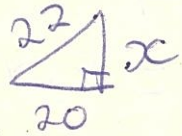
$b = 8.3$

$\boxed{CD = 16.6cm}$

Example #4: A horizontal pipe has a circular cross section, with center O. Its radius is 22 cm. Water fills less than one-half of the pipe. The surface of the water AB is 40 cm wide. Determine the maximum depth of the water (CD).

radii = 22





$22^2 - 20^2 = b^2$

$484 - 400 = 84$

$b = 9.2$

9.2

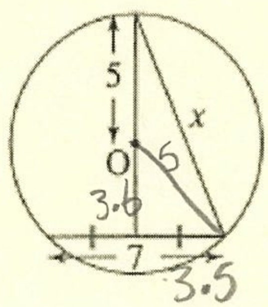
22

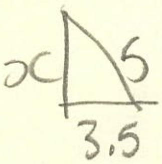
$22 - 9.2 = $ Depth of H_2O $\boxed{12.8cm}$

Example #5:

. Determine each value of x. Point O is the centre of each circle.

a)





$5^2 - 3.5 = b^2$

$25 - 12.25 = 12.75$

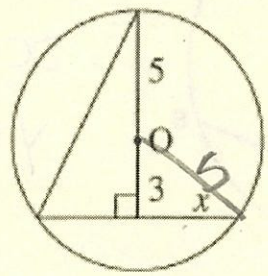
$b = 3.6$

$73.96 + 12.25 = c^2$

86.21

$c = 9.3$

b)



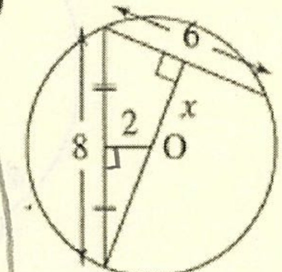
$5^2 - 3^2 = b^2$

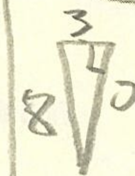
$25 - 9 = b^2$

$b^2 = 16$

$b = 4$

c)



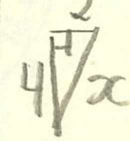


$8^2 - 3^2 = b^2$

$64 - 9 = b^2$

$b^2 = 55$

$b = 7.4$



$2^2 + 4^2 = c^2$

$4 + 16 = c^2$

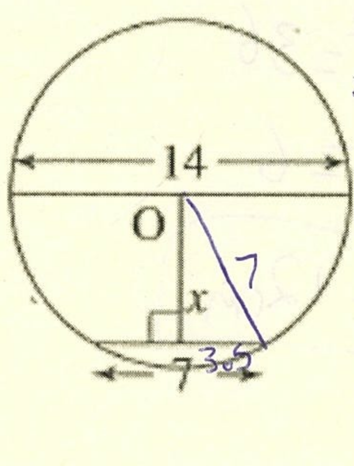
$20 = c^2$

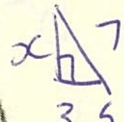
$c = 4.5$

Your turn, try these…

1. A circle (below left) has diameter 14 cm. A chord is 7 cm long. How far from the centre of the circle is the chord?

radii = 7





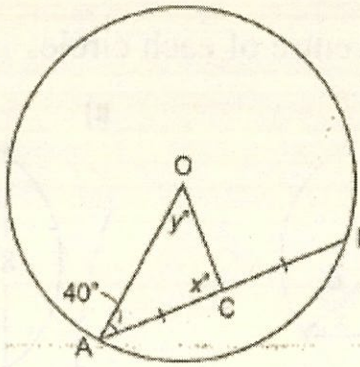
$7^2 - 3.5^2 = a^2$

$49 - 12.25 = a^2$

$a^2 = 36.75$

$\boxed{a = 6.06 \text{ cm}}$

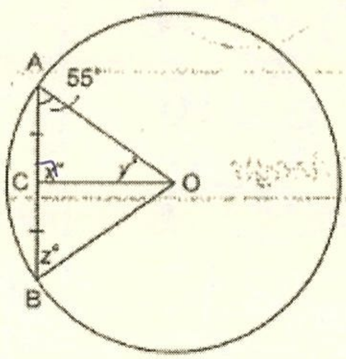
2. Find the values of $x°$ and $y°$.



$x = 90°$

$y = 50°$

3. Find the values of $x°$, $y°$, and $z°$.

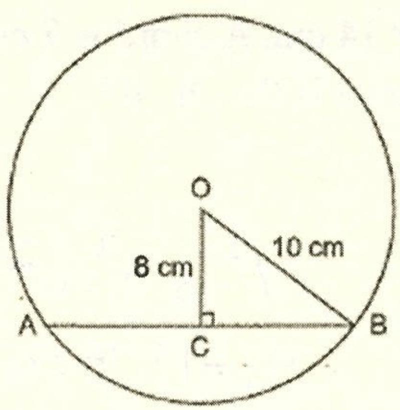


$x = 90$

$y = 35$

$z = 55$

4. O is the centre of the circle.
Find the length of chord AB.



$10^2 - 8^2 = b^2$

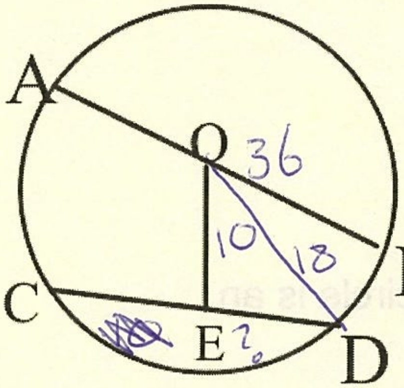
$100 - 64 = b^2$

$b^2 = 36$

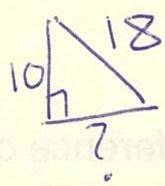
$b = 6$

$\boxed{\overline{AB} = 12\text{cm}}$

5. AB is a diameter with length 36 cm. CD is a chord that is 10 cm from the center O. What is the length of the chord?



radii = 18



$18^2 - 10^2 = b^2$

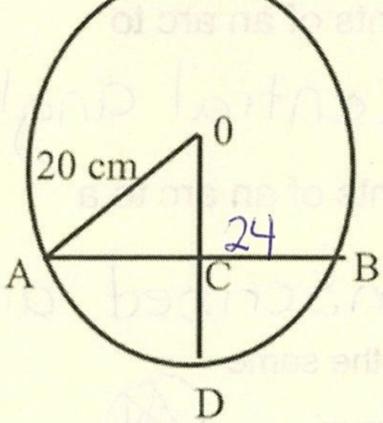
$324 - 100 = b^2$

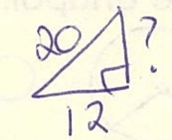
$b^2 = 224$

$b = 14.96$

$\boxed{CD = 29.93}$

6. A horizontal pipe has a circular cross section with center O. It has a radius of 20 cm. Water fills less than one-half of the pipe. The surface of the water AB is 24 cm wide. Determine the maximum depth of the water which is the depth CD.





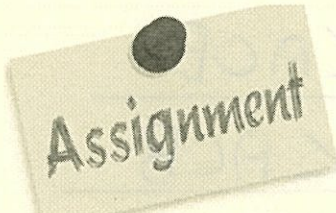
$20^2 - 12^2 = b^2$

$400 - 144 = b^2$

$b^2 = 256$

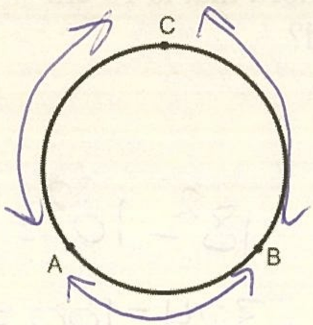
$b = 16$

$\boxed{20 - 16 = 4cm}$



Chord properties worksheet

9.3 Angles and arcs in a Circle



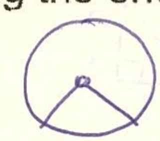
A section of the circumference of a circle is an

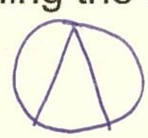
The shorter arc (less than 180 degrees) is a

Minor arc

The longer arc (greater than 180 degrees) is a

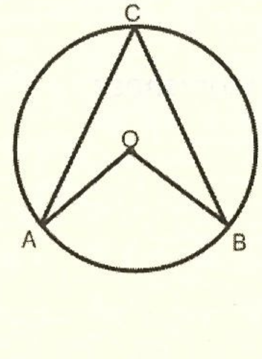
Major arc

The angle formed by joining the endpoints of an arc to the center of the circle is a  Central angle

The angle formed by joining the endpoints of an arc to a point on the circle is an  Inscribed angle

Key: Inscribed angles with base points from the same arc will always be Congruent. 

We can now say that the central angle and inscribed angle in the diagram are subtended by the Minor Arc AB.



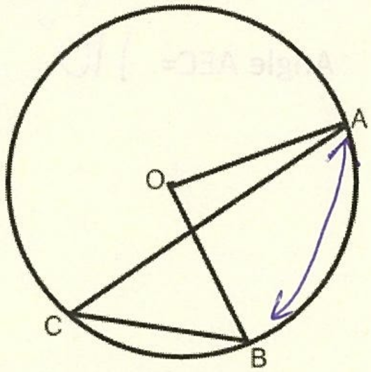
Central Angle = $\angle AOB$

Inscribed Angle = $\angle ACB$

Property #1:

An inscribed angle and a central angle subtending the same arc follow this rule:

Inscribed angle: $\frac{1}{2}$ the central angle size (X)
Central angle: 2x the inscribed angle (2X)



Identify the Central Angle: <BOA

Identify the Inscribed Angle: <BCA

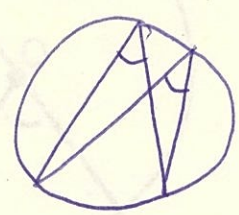
These angles are subtended by the arc BA

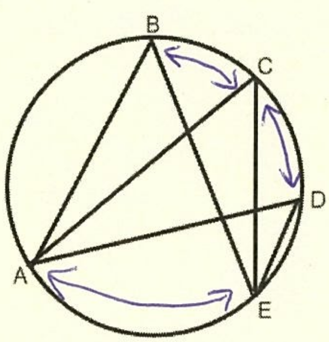
*** <OAC is NOT an inscribed because the sides do not touch the edge of the circle.

Property #2:
Inscribed angles from the same arc look like a bow-tie in many situations.

<u>Inscribed Angle Property</u>

In a circle, ALL inscribed angles subtended by the same arc are congruent.





Using $\overset{\frown}{AE}$ as the arc identify the congruent angles. (Use your fingers to trace if it makes it easier)

<ABE <ACE <ADE

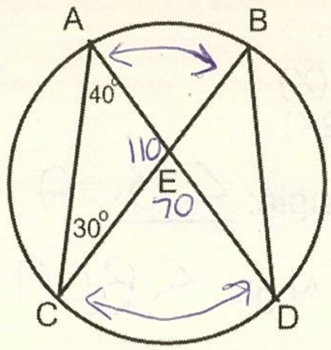
Using $\overset{\frown}{BC}$ as the arc identify the congruent angles.

<CAB <BEC

Using $\overset{\frown}{CD}$ as the arc identify the congruent angles.

<CAD <CED

Bow-tie rule:



Angle B= 40°

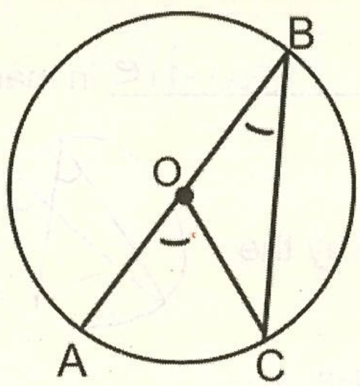
Angle CED= 70°

Angle D = 30°

Angle AEC= 110°

Angle E= 70°

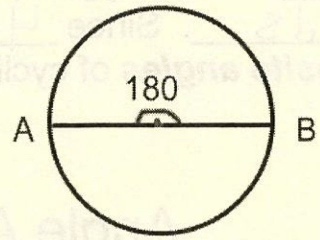
Something to think about….
Can an inscribed angle be formed from a diameter?



yes

<ABC

The two arcs formed by the endpoints of a diameter are semicircles.

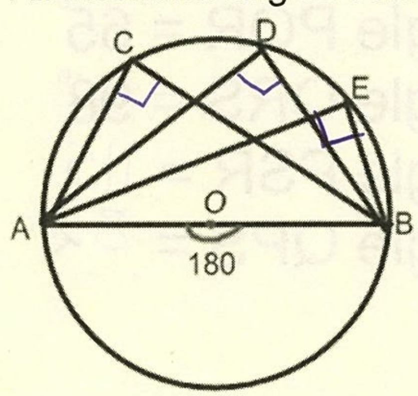


Supplementary angles
→ add to 180°

The central angle of each arc is a straight angle which measures 180 degrees.

Property #3:

All inscribed angles subtended by a semicircle are __right__ angles.

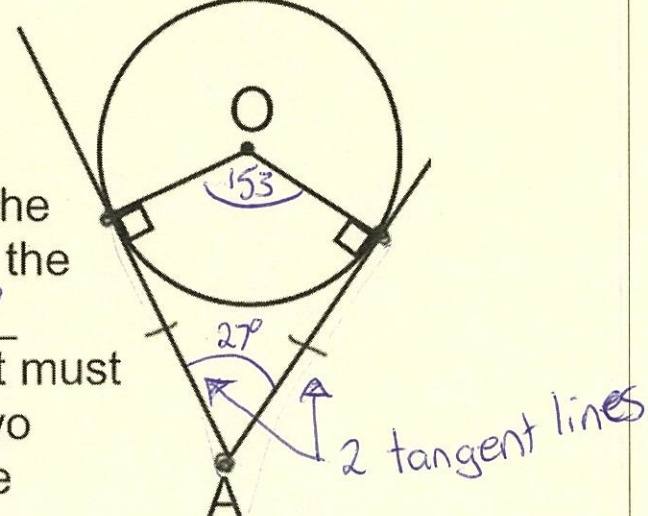


Identify the right angles in the diagram.

90°

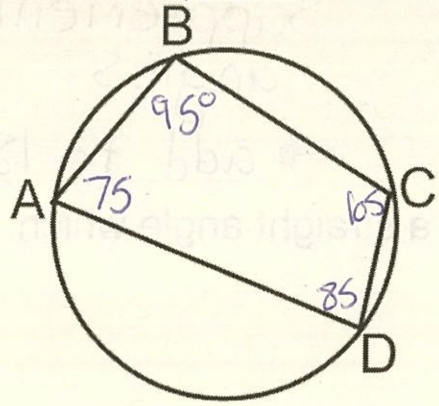
Property #4:

The combination of tangent & angle properties, when two tangent lines meet two radii there is now a 4-sided object formed with two 90^{o} angles at the points of tangency. Because of this, the other two angles must add to __180°__ since the degrees in a 4-sided object must add up to a total of __360°__. The two tangent lines are equidistant from the point, A, outside the circle.

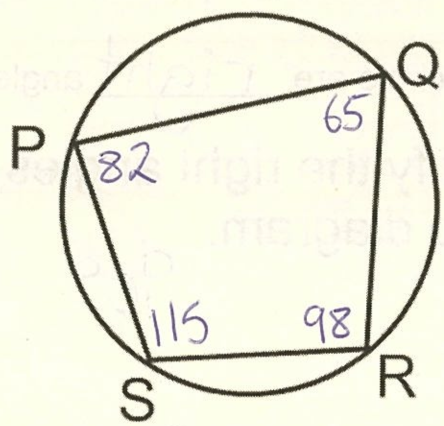


Property #5:

A ***cyclic quadrilateral*** is a 4-sided polygon (object) inside a circle created by 4 Chords. Since 4-sided polygons have a total of 360°, ***opposite angles*** of cyclic quadrilaterals MUST add to 180°.



Angle ABC = 95°
Angle BAD = 75°
Angle ADC = 85
Angle BCD = 105

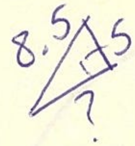


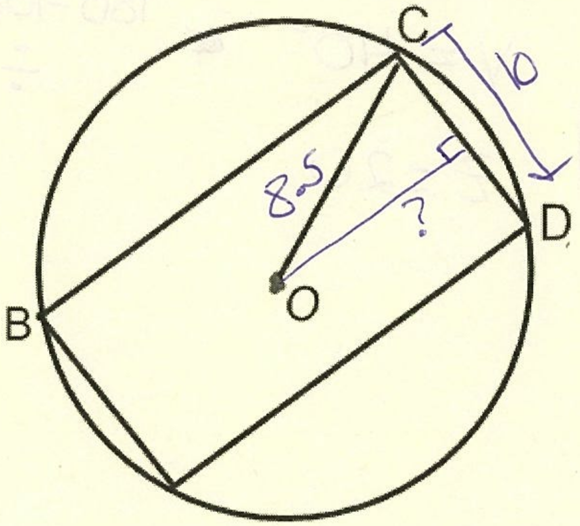
Angle PQR = 65°
Angle QRS = 98°
Angle PSR = 115
Angle QPS = 82

Applying all the properties – Let's try these together…

Example 1:

Rectangle ABCD has its vertices on a circle with radius 8.5 cm. The width of the rectangle is 10 cm. What is its length to the nearest tenth?





$8.5^2 - 5^2 = ?^2$

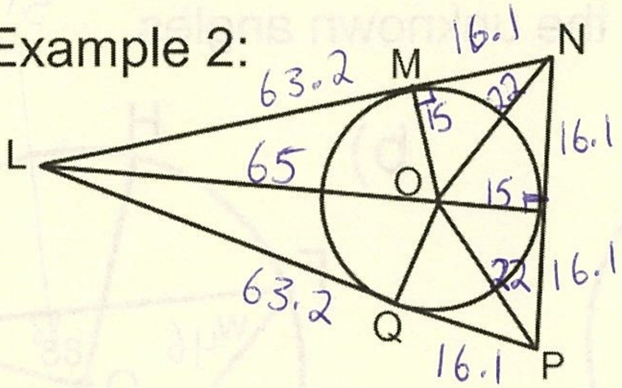
$72.25 - 25 = b^2$

$47.25 = b^2$

$b = 6.87$

$\boxed{\times 2 = 13.7\text{cm}}$

Example 2:



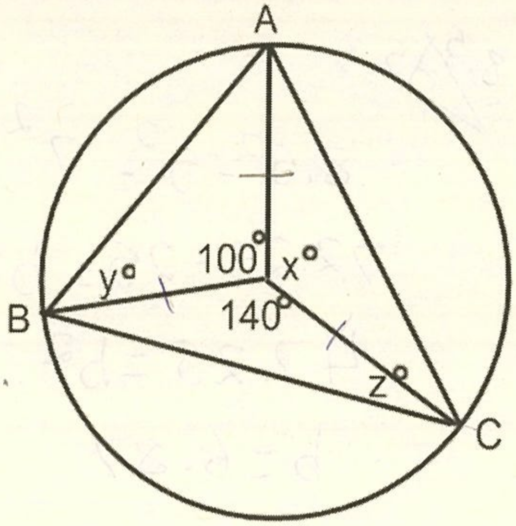
Radius = 15 cm
OP = 22 cm
OL = 65 cm

Find the perimeter of triangle LNP.

$\boxed{190.9\text{cm}}$

Example 3:

Triangle ABC is inscribed in a circle with center O.
<AOB = 100° and <COB = 140°
Determine the values of x°, y°, z° and angle BAC.



$x = 120°$

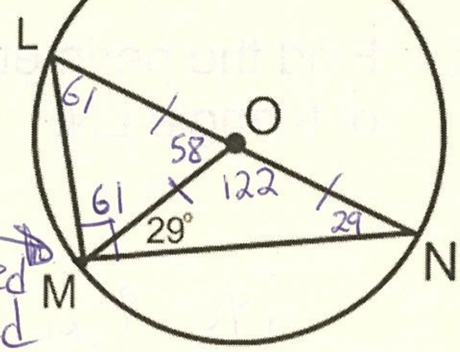
$y = 40°$ — $180 - 100 = 80$ $\div 2 = 40$

$z = 20$

Example 4: Solve all the unknown angles.

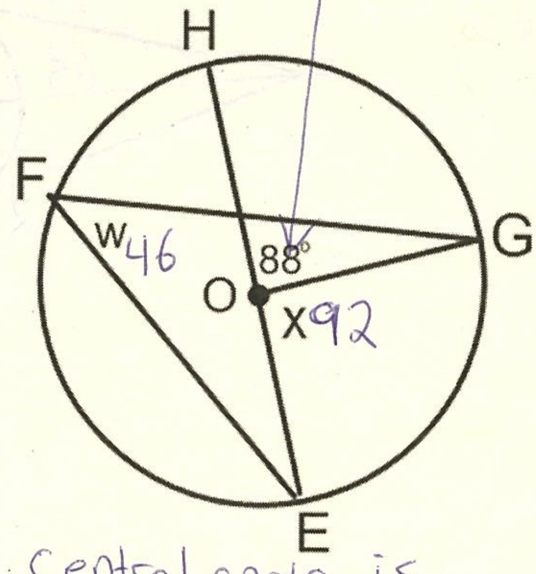
straight line

a)



inscribed Subtended by diameter is 90°

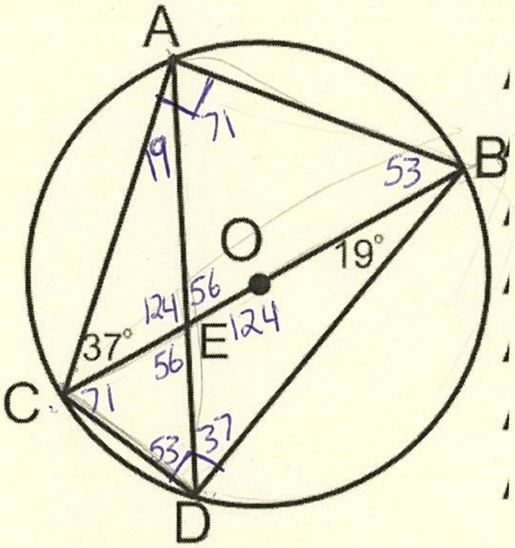
b)

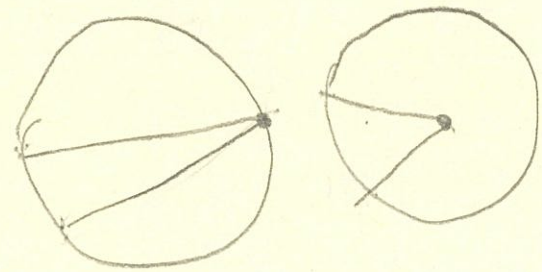


Central angle is double inscribed Subtended by same arc.

*Bow-tie rule AND inscribed angles subtended by diameter=90°

c)





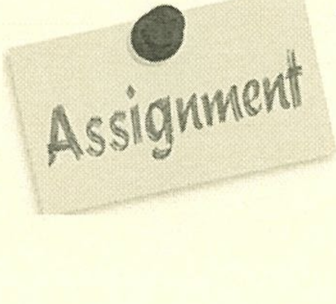
inscribed subtending diameter (90°)

Angle CAD = 19
Angle ADB = 37
Angle DEB = 124
Angle AEB = 56
Angle AEC = 124
Angle CED = 56
Angle ECD = 71

Do EDC first

Angle EDC = 53
Angle EAB = 71
Angle EBA = 53
Angle CDB = 90
Angle CAB = 90
Angle ACD = 108
Angle DBA = 180 - 108 = 72°

opposite angles in cyclic quad add to 180°



Arc and angles properties worksheet